

**Sec 4.4.3**  
**Solving Logarithmic Equations**

$$\log_5 x = 3$$

$$5^3 = x$$

$$125 = x$$

$$\log_2(4x+1) = 5$$

$$2^5 = 4x+1$$

$$32 = 4x+1$$

$$\frac{31}{4} = x$$

log = number ----> rewrite as a power

$$7 + 3 \ln x = 6$$

$$3 \ln x = -1$$

$$\ln x = -\frac{1}{3}$$

$$e^{-\frac{1}{3}} = x \rightsquigarrow \text{exact}$$

$$\bullet 0.717 \approx$$

1. Isolate log/ln
2. Rewrite as a power
3. Solve resulting equation.

$$\ln\sqrt{x+4} = 1$$

$$e = \sqrt{x+4}$$

$$e^2 = x+4$$

$$e^2 - 4 = x \quad \leftarrow \text{exact}$$

$$3.389 \approx \quad \leftarrow \text{approx}$$

exact and approximate...

1. Isolate log/ln
2. Rewrite
3. Solve
4. Check for extraneous

$$\log_3(x+6) + \log_3(x+4) = 1$$

$$\log_3(x+6)(x+4) = 1$$

$$3 = x^2 + 10x + 24$$

$$0 = x^2 + 10x + 21$$

$$0 = (x+7)(x+3)$$

Remember...you cannot take the log/ln of a negative or zero...

$$x = \cancel{7} - 3 \leftarrow \text{only}$$

1. Condense to a single log
2. Rewrite
3. Solve
4. Check

$$\log_4(x+2) - \log_4(x-1) = 1$$

$$\log_4 \frac{x+2}{x-1} = 1$$

$$4 = \frac{x+2}{x-1}$$

$$4(x-1) = x+2$$

$$4x - 4 = x + 2$$

$$3x = 6$$

$$x = 2$$

$$\log(3x-3) = \log(x+1) + \log 4$$

$$\log(3x-3) = \log 4(x+1)$$

$$3x-3 = 4x+4$$

$$~~-7 = x~~$$

no  
solutions

Logs on  
both sides?  
Condense  
to one on  
each, then  
set  
arguments  
equal.

**Suggested Practice  
Sec 4.4 page 490  
49-92  
every other odd**

**These problems have various  
nuances...small changes  
throughout. I suggest trying  
every suggested, and  
especially several from  
83-92.**

49-92 every other odd...

1. [6]    3. [3]    5. [3]    7. [2]    9.  $\left\{\frac{3}{5}\right\}$     11.  $\left\{\frac{3}{2}\right\}$     13. [4]    15. [5]    17.  $\left\{-\frac{1}{4}\right\}$     19. [13]    21. [-2]  
 23.  $\left\{\frac{\ln 3.91}{\ln 10}\right\}; \approx 0.59$     25.  $\{\ln 5.7\}; \approx 1.74$     27.  $\left\{\frac{\ln 17}{\ln 5}\right\}; \approx 1.76$     29.  $\left\{\ln \frac{23}{5}\right\}; \approx 1.53$     31.  $\left\{\frac{\ln 659}{5}\right\}; \approx 1.30$   
 33.  $\left\{\frac{\ln 793 - 1}{-5}\right\}; \approx -1.14$     35.  $\left\{\frac{\ln 10,478 + 3}{5}\right\}; \approx 2.45$     37.  $\left\{\frac{\ln 410}{\ln 7} - 2\right\}; \approx 1.09$     39.  $\left\{\frac{\ln 813}{0.3 \ln 7}\right\}; \approx 11.48$     41.  $\left\{\frac{3 \ln 5 + \ln 3}{\ln 3 - 2 \ln 5}\right\}; \approx -2.80$   
 43.  $[0, \ln 2]; \ln 2 \approx 0.69$     45.  $\left\{\frac{\ln 3}{2}\right\}; \approx 0.55$     47. [0]    49. [81]    51.  $\{e^2\}; \approx 7.39$     53. [59]    55. [-9]    57.  $\left\{\frac{107}{27}\right\}$     59.  $\left\{\frac{62}{3}\right\}$   
 61.  $\left\{\frac{e^4}{2}\right\}; \approx 27.30$     63.  $\{e^{-1/2}\}; \approx 0.61$     65.  $\{e^2 - 3\}; \approx 4.39$     67.  $\left\{\frac{5}{4}\right\}$     69. [-3]    71. [6]    73. [5]    75. [12]    77.  $\left\{\frac{4}{3}\right\}$     79.  $\emptyset$   
 81. [5]    83.  $\left\{\frac{2}{9}\right\}$     85. [28]    87. [2]    89.  $\emptyset$     91.  $\left\{\frac{11}{3}\right\}$     93.  $\left\{\frac{1}{2}\right\}$     95.  $\{e^3, e^{-3}\}$     97.  $\left\{\pm \sqrt{\frac{\ln 45}{\ln 3}}\right\}$     99.  $\left\{\frac{5 + \sqrt{37}}{2}\right\}$