

Sec 3.5.4
Finding slant asymptotes...
....and graphing

A slant asymptote only exists if the degree of the numerator is exactly ONE more than the degree of the denominator.

(note, a horizontal asymptote will not exist)

We will determine them using long or synthetic division.

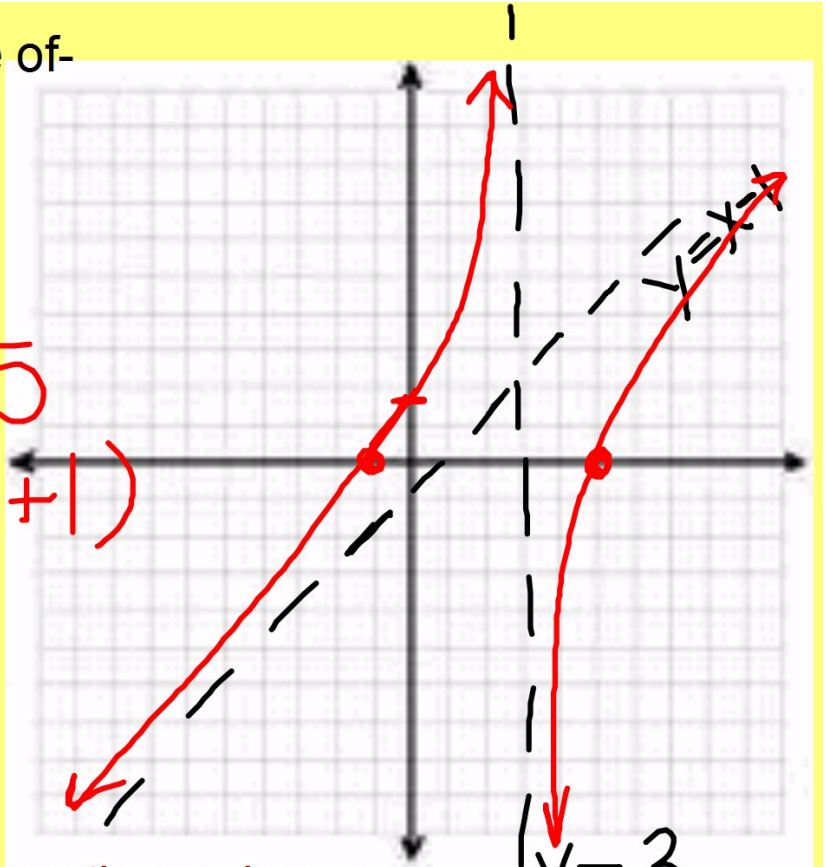
A graph can cross a
slant and horizontal asymptote.
A graph cannot cross a vertical
asymptote, ever, because
these are "bad spots" in the
domain.

Find the slant asymptote of-

(to graph, also find all intercepts)

$$f(x) = \frac{x^2 - 4x - 5}{x - 3}$$

$$\begin{aligned} 0 &= x^2 - 4x - 5 \\ &= (x - 5)(x + 1) \\ x &= 5 \text{ \& } -1 \end{aligned}$$



*degree in numerator is exactly one larger

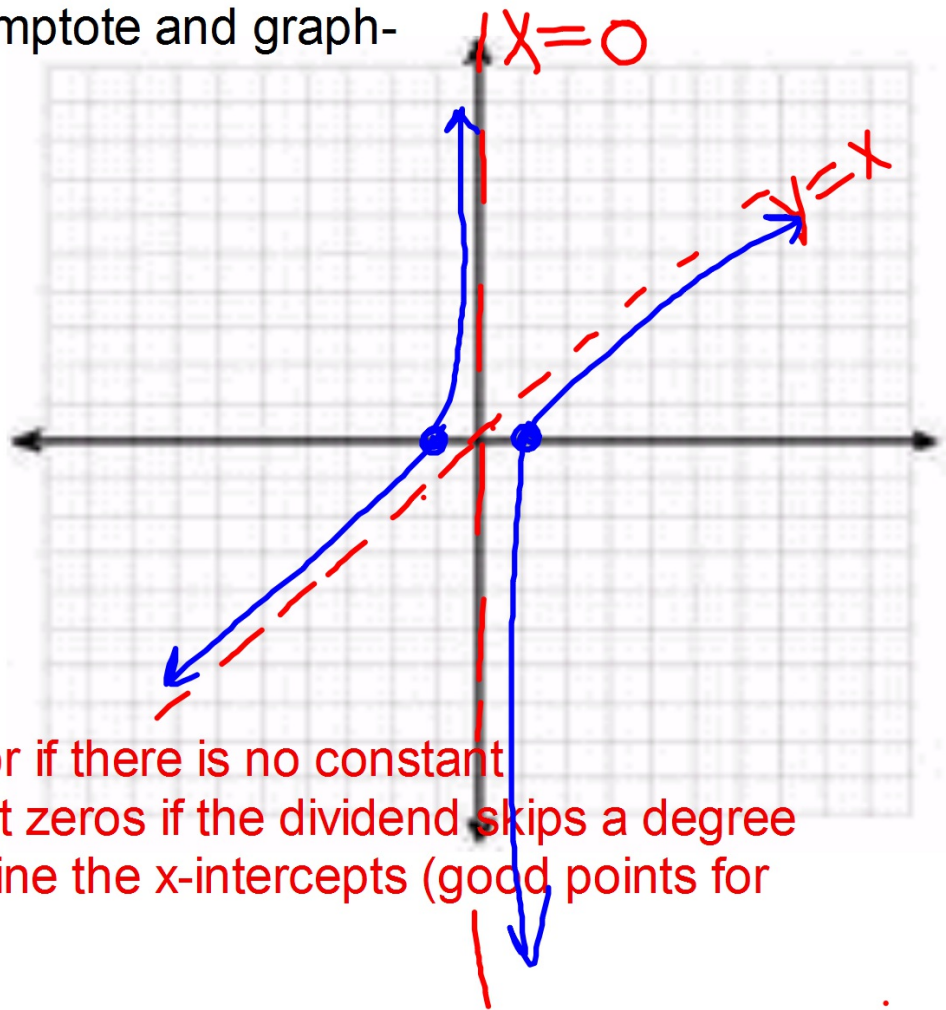
*ignore remainder when writing the equation for the SA

Find the slant asymptote and graph-

$$(x+1)(x-1)$$

$$f(x) = \frac{x^2 - 1}{x + 0}$$

$$0 = x^2 - 1$$
$$1 = x^2$$



- *use 0 as the divisor if there is no constant
- *remember to insert zeros if the dividend skips a degree
- *let $y = 0$ to determine the x-intercepts (good points for this function)

Graph-

$$f(x) = \frac{x^3}{x^2-1} = \frac{x^3}{(x+1)(x-1)}$$

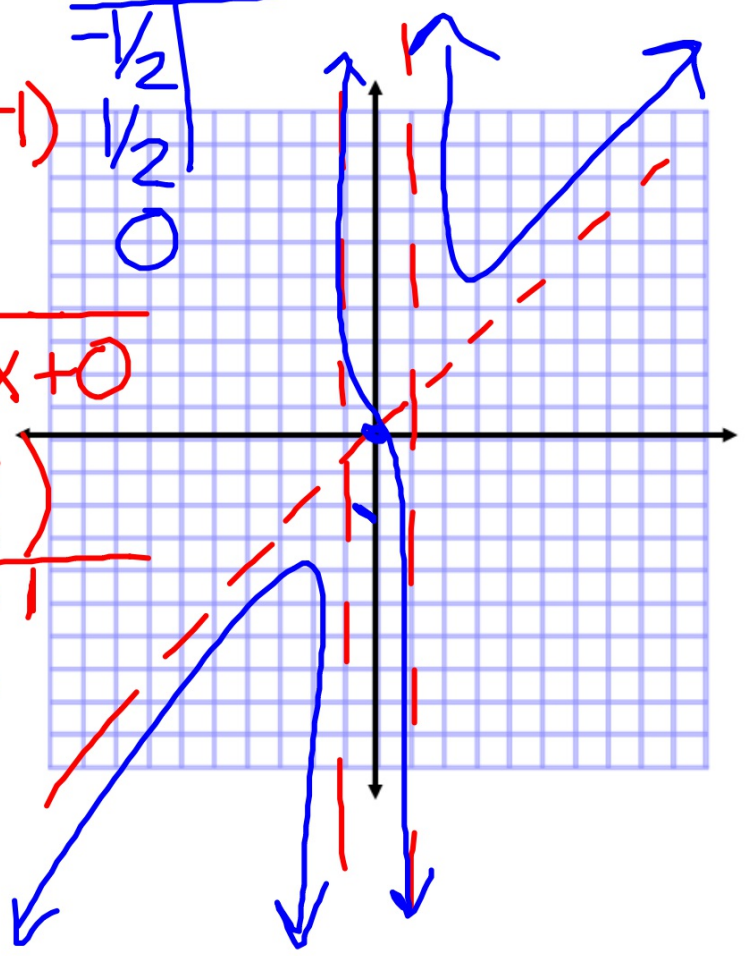
$$y = x$$

$$x^2-1 \overline{) x^3 + 0x^2 + 0x + 0}$$

$$\underline{-(x^3 - x)} \\ x$$

$$0 = \frac{0}{0-1}$$

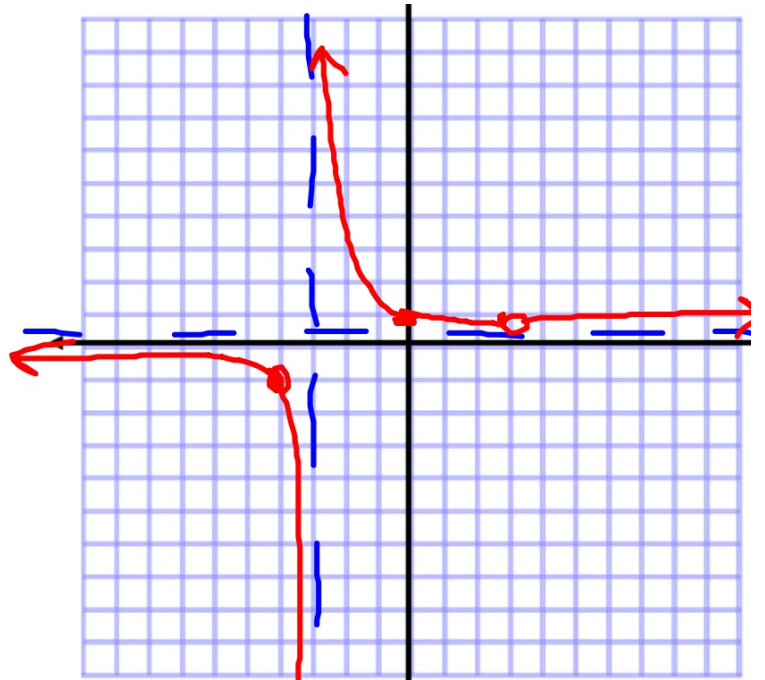
$-\frac{1}{2}$
$\frac{1}{2}$
0



Graph $f(x) = \frac{x-3}{x^2-9}$

$$\begin{aligned} & \frac{x-3}{(x+3)(x-3)} \\ &= \frac{1}{x+3} \end{aligned}$$

x	y
0	1/3
-5	-1/2



HA @ $y=0$
VA @ $x=-3$

hole @ $x=3$

Suggested Practice

Sec 3.5
page 407
73,83,85,87

When graphing, show work for slant asymptotes and intercepts. Clearly label a point on each side of every vertical asymptote (or have a table). Test points and other details can be gleaned from the graphing calculator.

73.

VA @ $x = -2$

HA @ $y = 0$

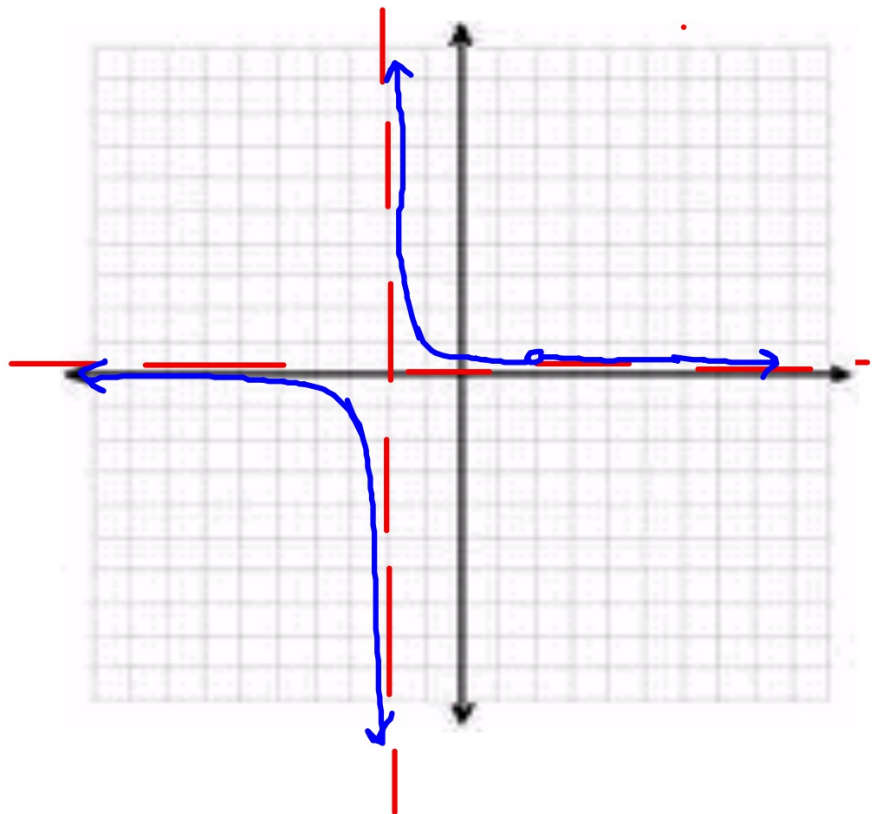
hole at $x = 2$

y-int: $(0, \frac{1}{2})$

other points-

$(-3, -1)$

$(-1, 1)$



83.

VA @ $x = 0$

no HA

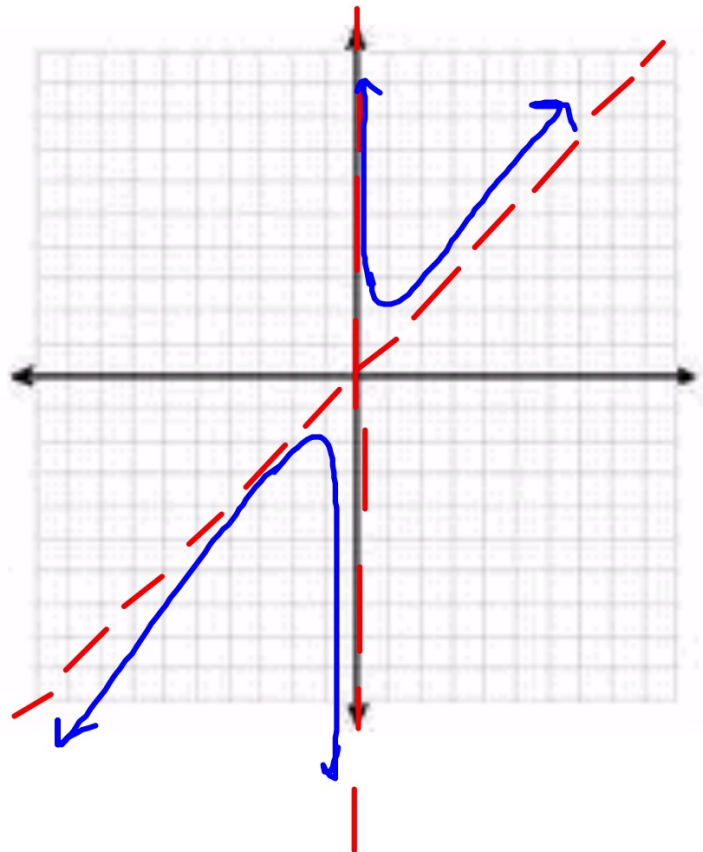
SA @ $y = x$

other points:

(1,2)

(-1,-2)

no intercepts



85.

VA @ $x = 3$

no HA

SA at $y = x + 4$

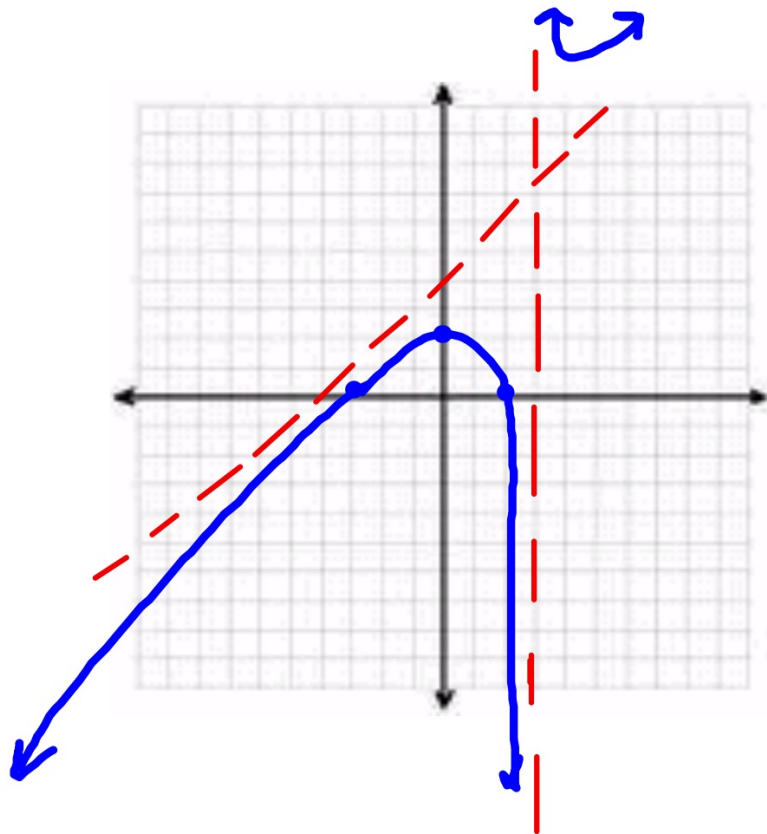
y-int @ $(0, 2)$

x-int @ $(-3, 0)$

$(2, 0)$

other point-

$(4, 14)$



87.

VA @ $x = 0$

$x = -2$

SA @ $y = x - 2$

no y-intercepts

x-intercept

@ $(-1, 0)$

other points:

$(1, 2/3)$

$(-4, -63/8)$

