Sec 3.4.4
Descartes' Rule of Signs

put this on your card

We can determine the number of positive real zeros by looking at SIGN CHANGES.

Descartes' Rule of Signs

1. The number of possible positive real zeros is equal to the number of changes in the sign of the coefficients of $f(x)$ or less by an even integer.
2. The number of possible negative real zeros is equal to the number of changes in the sign of the coefficients of $f(-x)$ or less by an even integer.

$f(x) = 3x^7 - 2x^5 - x^4 + 7x^2 + x - 3$

sign change

3 sign changes

$\therefore$ 3 positive real zeros or ←

$3 - 2 = 1$ positive real zeros ←

↑ even #

3 or 1 possible positive reals.

$f(x) = 4x^5 + 2x^4 - 3x^2 + x + 5$

2 sign changes

2 or 0

$\therefore$ 2 positive real zeros

or $2 - 2 = 0$ positive real zeros

2 ← even

$f(x) = -7x^6 - 5x^4 + x + 9$

1 sign change

$\therefore$ 1 positive real zero } only possibility

or 1-2 ~~or~~

$$f(x) = x^5 + 3x^4 - 2x^3 - x^2 + x - 3$$

1 2 3

# of possible positive real zeros

3 or 1

To determine the possible number of *negative* real zeros...

- **replace x with -x**
- **"clean it up" (e.g. raise to power, multiply negatives)**
- **count the sign changes**

For example- $f(x) = x^3 + 2x^2 + 5x + 4$

$f(-x) = (-x)^3 + 2(-x)^2 + 5(-x) + 4$

$= -x^3 + 2x^2 - 5x + 4$

so $f(-x) = -x^3 + 2x^2 - 5x + 4$

There are three sign changes, so either three or 1 negative real zeros.

1 2 3

$$f(x) = 6x^5 - 4x^3 - 3x^2 + 5$$

$$= 6(-x)^5 - 4(-x)^3 - 3(-x)^2 + 5$$

$$= -(6x^5 + 4x^3 - 3x^2 + 5)$$

3 sign changes

→ 3 or 1 possible negative real

Put one (or all three) examples on your notecard...

Try...
Sec 3.4
page 387
determine the possible number of real zeros
#33,35,37

and...
find all zeros of 39, 43

33. no positive, 3 or 1 negative real roots
35. 3 or 1 positive, no negative
37. 2 or 0 positive, 2 or 0 negative
39. -2,5,1
43. -1, 2+2i, 2-2i

ex $f(x) = 4x^5 + 3x^3 - 6x^2 - 7x - 2$

Positive = 1

$f(-x) = 4(-x)^5 + 3(-x)^3 - 6(-x)^2 - 7(-x) - 2$

$= -4x^5 - 3x^3 - 6x^2 + 7x - 2$

Negative = 2, 0