

Sec 3.4.1  
Determining the Zeros of Polynomial Functions

List the possible rational zeros-  $\frac{\text{factors of the constant}}{\text{factors of the leading coefficient}}$

$$f(x) = x^3 + 2x + 6$$

$$g(x) = -4x^3 - 3x + 10$$

$$\begin{array}{l} C \rightarrow 10, 1, 5, 2 \\ \hline LC \rightarrow 4, 1, 2 \end{array}$$

$$\text{Possible} \rightarrow \pm \frac{5}{2}, 10, 5, \frac{1}{4}, 1, \frac{1}{2}, \frac{5}{4}, 2,$$

\*this is called the rational zero theorem

Finding all zeros-

Find all zeros of  $f(x) = x^3 + 2x^2 - 5x - 6$

$$C \rightarrow \pm 6, \pm 3, \pm 2, \pm 1 \quad \leftarrow \text{possible}$$

$$LC \rightarrow \pm 1$$

$$\begin{array}{r} 2 \\ | \end{array} \left| \begin{array}{cccc} 1 & 2 & -5 & -6 \\ & 2 & 8 & 6 \\ & \hline 1 & 4 & 3 & 0 \end{array} \right.$$

Coefficients  
of quotient

$$(x-2)(x^2+4x+3)=0$$

$$(x-2)(x+3)(x+1)=0$$

$$x = \{-3, -1, 2\}$$

Find all zeros of  $f(x) = x^3 + 7x^2 + 11x - 3$

C  $\rightarrow \pm 3, 1$  ← possible

L C  $\rightarrow \pm 1$

$$\begin{array}{r} -3 \\ \hline 1 & 7 & 11 & -3 \\ & -3 & -12 & 3 \\ \hline 1 & 4 & -1 & 0 \end{array}$$

$$(x+3)(x^2+4x-1)=0$$

$$x = \frac{-4 \pm \sqrt{16 - 4(-1)}}{2}$$

$$= \frac{-4 \pm \sqrt{20}}{2} \text{ or } 4.5$$

$$= \frac{-4 \pm 2\sqrt{5}}{2}$$

$$= -2 \pm \sqrt{5}$$

$$x = \{-3, -2+\sqrt{5}, -2-\sqrt{5}\}$$

Find all zeros of  $f(x) = x^3 - 4x^2 + 8x - 5$

$$C \rightarrow \frac{\pm 5 \pm 1}{\pm 1} \leftarrow \text{possibles}$$

$$\begin{array}{r|rrrr} & 1 & -4 & 8 & -5 \\ & & 1 & -3 & 5 \\ \hline & 1 & -3 & 5 & | 0 \end{array}$$

$$(x-1)(x^2-3x+5)=0$$

$$x = \frac{3 \pm \sqrt{9 - 4(5)}}{2}$$

$$= \frac{3 \pm \sqrt{-11}}{2} \quad ^{0.00-1.011}$$

$$= \left\{ \frac{3 \pm \sqrt{-11}}{2}, 1 \right\}$$



**Suggested Practice**

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9, 11, 13, 15**

9. a.  $\pm 1, \pm 2, \pm 4$   
b.  $-2, -1, 2$   
c.  $-2, -1 \notin 2$

13. a.  $\pm 1, \pm 2, \pm 3, \pm 6$   
b.  $-1$

11. a.  $\pm 1, \pm 2, \pm 3, \pm 6, \frac{1}{2}, \frac{3}{2}$

c. also  
$$\frac{-3 \pm \sqrt{33}}{2}$$

b.  $-2, \frac{1}{2}, 3$

c. same

15. a.  $\pm 1, \pm \frac{1}{2}, \pm 2$   
b.  $-2$   
c. and...  $\frac{-1 \pm i}{2}$