Sec 3.3.3
Using division to determine zeros
(roots, real solutions)

First, the Remainder Theorem states...if the polynomial f(x) is divided by x -c, then the remainder is f(c).

i.e. You can evalute a function using synthetic division. The remainder is the "y"/function value.

Use the remainder theorem to evaluate

$f(x) = x^3 - 4x^2 + 5x + 3$ at $f(2)$

2	1	−4	5	3
		2	−4	2
	1	−2	1	5

So, $f(2) = 5$ ⟵ (2,5) point on the curve

The Factor Theorem

If there is no remainder, then the divisor is a factor (or, in the case of synthetic division, a "zero") of the dividend.

Example-

$$x+3 \overline{\smash{)}\,x^2+10x+21}$$

If there is no remainder, then x+3 is factor of $x^2+10x+21$ and we know that $(x+7)(x+3) = x^2+10x+21$

$$2x^3-3x^2-11x+6 \div x-3$$

There is no remainder so we know that 3 is a zero/root/solution to $2x^3-3x^2-11x+6$ and that (x-3) is a factor of it.

Solve the equation $2x^3-3x^2-11x + 6 = 0$ given that 3 is a zero/root.

$$\begin{array}{c|cccc} 3 & 2 & -3 & -11 & 6 \\ & & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & |\,0 \end{array}$$

$$(x-3)(2x^2+3x-2)=0$$

$$x^2+3x-4$$

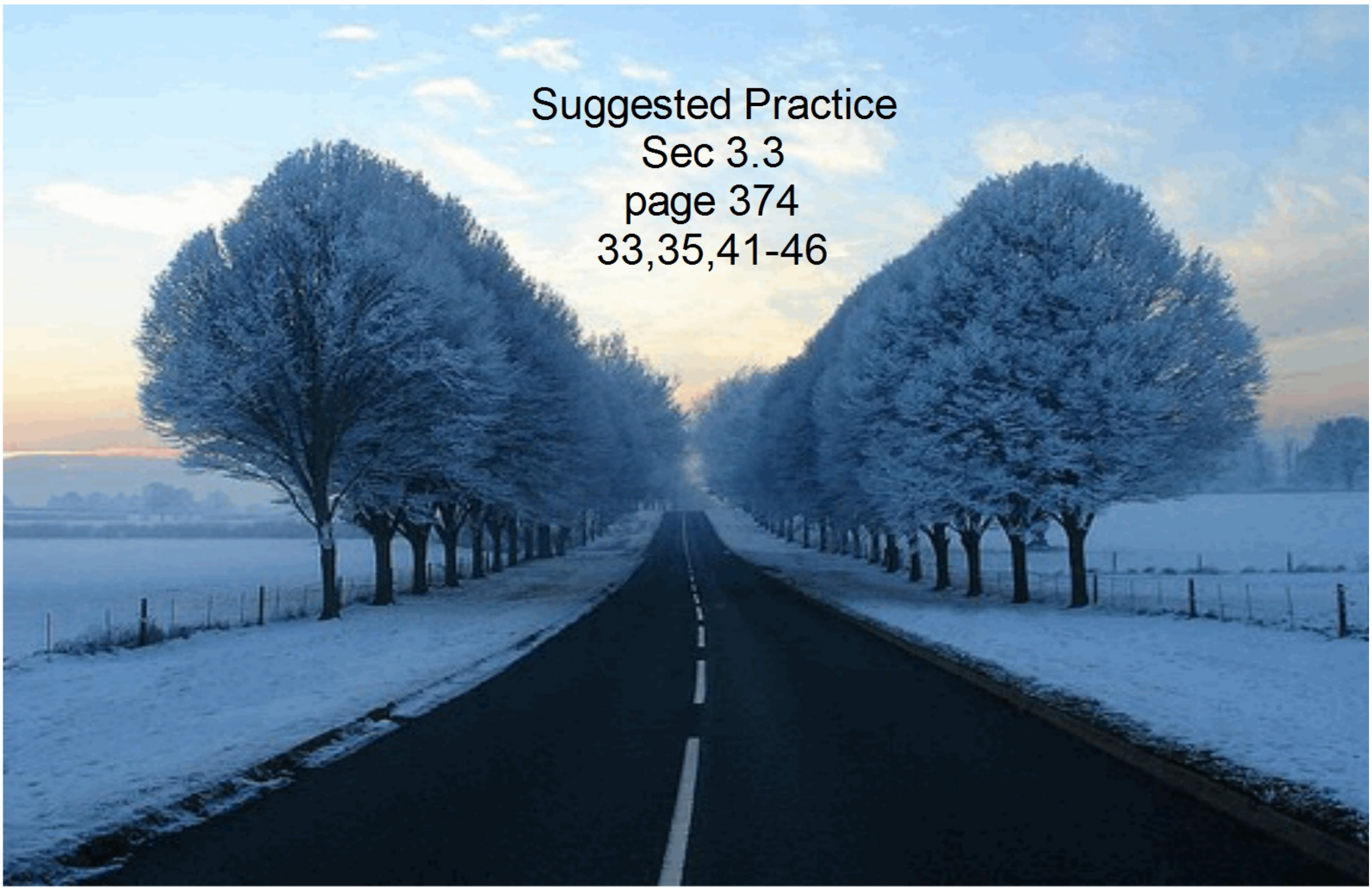
$$(x+\tfrac{4}{2})(x-\tfrac{1}{2})$$

$$(x-3)(x+2)(2x-1)=0$$

$$x+2=0 \quad 2x-1=0$$

$$x=-2 \quad x=1/2$$

$\{-2, 1/2\}$


Suggested Practice
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33. -25

35. -133

41. x^2-5x+6

$x=-1,2,3$

42. x^2-3x+2

$x=\pm 1,2$

43. $\{-1/2, 1, 2\}$

44. $\{-2, 1/2, 3\}$

45. $\{-3/2, -1/3, 1/2\}$

46. $\{-4, -1/3, 2\}$