Sec 2.7.1

Inverses

Definition-

informal- two functions that "undo" or "reverse" each other.

formal- Let f and g be two functions such that

$f(g(x)) = x$ for every x in the domain of g

and $g(f(x)) = x$ for every x in the domain of f.

i.e. the composite of two inverses, in both directions, yields only "x", the original input value

Notation-

$f^{-1}(x) =$

THE –1 NOTATION IS A BIT CONFUSING–

FOR FUNCTIONS F(X), SIN X, CSC X
IT INDICATES AN "INVERSE FUNCTION"

ON A NUMBER OR VARIABLE THE SAME NOTATION INDICATES A RECIPROCAL (E.G. 2^{-3})

Verify that $f(x) = 3x + 2$ and $g(x) = \frac{x-2}{3}$ are inverses.

$$f \circ g(x) = 3\left(\frac{x-2}{3}\right) + 2$$

$$= x - 2 + 2$$

$$= x$$

$$g \circ f(x) = \frac{3x - 2 + 2}{3}$$

$$= \frac{3x}{3} = x$$

b/c $g \circ f(x) = x$ & $f \circ g(x) = x$ we know $f(x)$ & $g(x)$ are inverses.

Verify that $f(x) = \frac{3}{x-4}$ and $g(x) = \frac{3}{x} + 4$ are inverses.

$$f \circ g(x) = \frac{3}{\frac{3}{x} + 4 - 4}$$

$$= \frac{3}{3/x} = 3\left(\frac{x}{3}\right) = x$$

$$g \circ f(x) = \frac{3}{\frac{3}{x-4}} + 4$$

$$= \cancel{3\left(\frac{x-4}{3}\right)} + 4$$

$$= x - 4 + 4 = x$$

Finding the inverse of a function-

1. Replace function notation with "y"
2. Interchange "x" and "y"
3. Solve for y

Find the inverse of f(x) = 7x-5

$$y = 7x - 5$$

$$x = 7y - 5$$

$$\frac{x+5}{7} = y \rightarrow \frac{1}{7}x + \frac{5}{7}$$

$$= f^{-1}(x)$$

Find the inverse of f(x) = x^3 +1

$$y = x^3 + 1$$

$$x = y^3 + 1$$

$$x - 1 = y^3$$

$$\sqrt[3]{x-1} = y = f^{-1}(x)$$

Find the inverse of $f(x) = \frac{5}{x} + 6$

$$y = \frac{5}{x} + 6$$

$$x = \frac{5}{y} + 6$$

$$\left[x - 6 = \frac{5}{y}\right] y$$

$$y(x-6) = 5$$

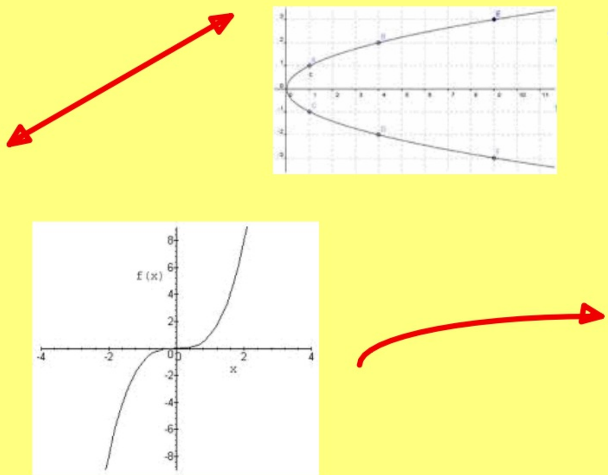
$$y = \frac{5}{x-6} = f^{-1}(x)$$

Determining, using a graph, if a function has an inverse...

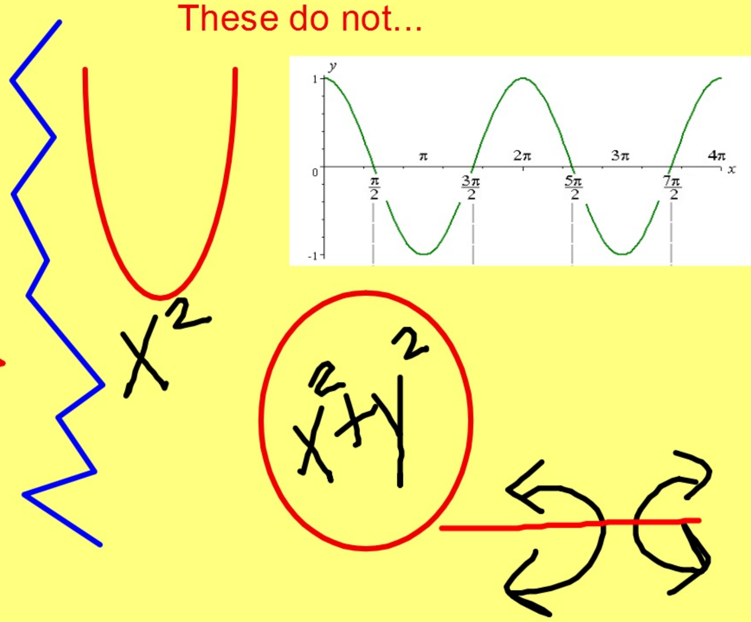
Use the Horzontal Line Test-

A function has an inverse if there is no horizontal line that intersects the graph of the function more than once.

These have inverses...

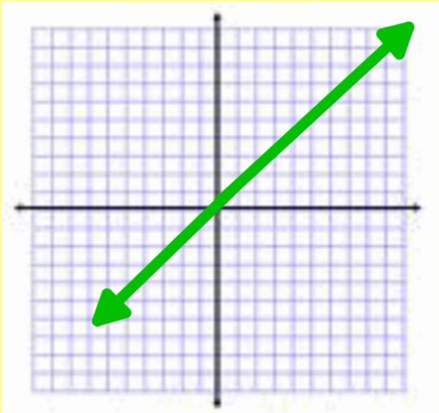


These do not...



Symmetry

Two functions which are inverses of each other, when graphs, will have symmetry with respect to the line y = x



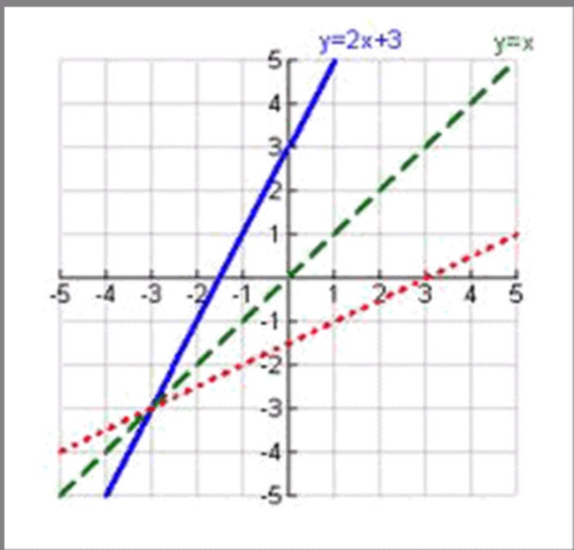
...and, their points are transposed. If (a, b) is on f, then (b,a) is on its inverse.

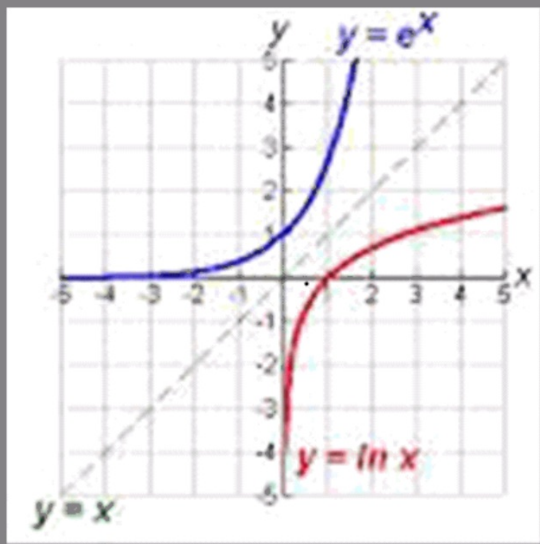
f (1,4)

f^{-1} (4,1)

The red and blue line are inverses. They're symmetric with respect to the green line, $y = x$

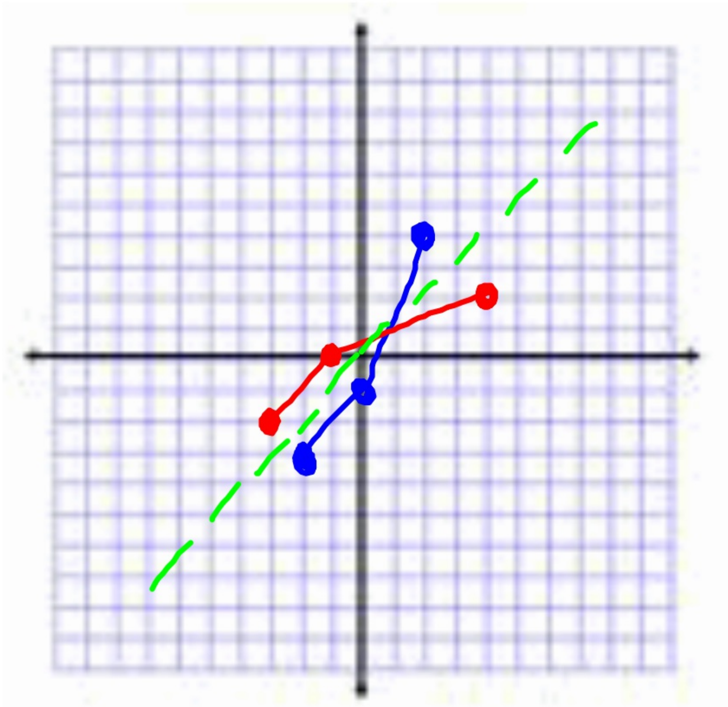
$y = e^x$ and $y = \ln x$ are inverses, as we know from solving equations. Here are their graphs:





Sketch the inverse of a function that contains (-3,-2), (-1,0) and (4,2). f

$(-2,-3)$
$(0,-1)$ f^{-1}
$(2,4)$



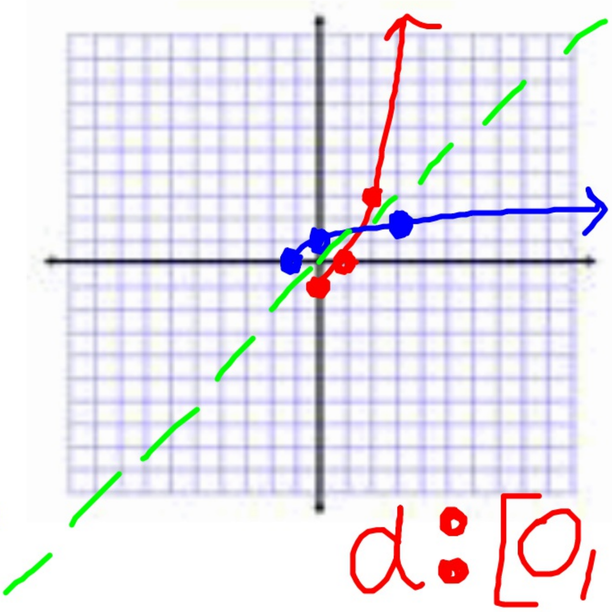
Find the inverse of $f(x) = x^2 - 1$ if $x \geq 0$ and graph each.

$$y = x^2 - 1$$

$$x = y^2 - 1$$

$$x + 1 = y^2$$

$$\sqrt{x+1} = y$$



$d: [0, \infty)$

$r: [-1, \infty)$

Determine the domain and range of each.

$$f^{-1}(x) = \sqrt{x+1}$$

$d: [-1, \infty)$ $r: [0, \infty)$

Suggested Practice-
Sec 2.7
page 309
1-27 odds
29-36
39-47 odds

1. are inverses
3. "
5. not inverses
7. are
9. are

11. $x-3$

13. $\frac{x}{2}$

15. $\frac{x-3}{2}$

17. $\sqrt[3]{x-2}$

19. $\sqrt[3]{x}-2$

21. $\frac{1}{x}$

23. x^2, $x \geq 0$
(to pass HLT)

25. $\dfrac{7}{x+3}$

27. $\dfrac{3x+1}{x-2}$

$\rightarrow (x \neq 2)$

HINT....SWITCH both y'S and use Factoring With #27....

29. no

30. yes

31. no

32. no

33. yes

34. yes

35. include points (-4,0), (0,2), (2,3)

36. include points (0,-3), (2,-1) & (3,5)

look @ sketch in key?

39. $\frac{x+1}{2}$

41. $\sqrt{x+4}$

43. $-\sqrt{x}+1$

45. $\sqrt[3]{x+1}$

47. $\sqrt[3]{x}-2$

d: $\mathbb{R}$

39. $f(x) = 2x - 1$ r: $\mathbb{R}$

$$x = 2y - 1$$

$$x + 1 = 2y$$

$$\frac{x+1}{2} = y = f^{-1}(x)$$

$$\frac{1}{2}x + \frac{1}{2}$$