Sec 1.6.4
Solving Equations in Quadratic Form
....that are not quadratic

$5x^{2/3} + 11x^{1/3} + 2 = 0 \dashrightarrow 5(x^{1/3})^2 + 11x^{1/3} + 2 = 0$

Rewrite-
Let $u = x^{1/3}$

$5u^2 + 11u + 2 = 0$

$u = x^{1/3}$

$\left(\frac{-1}{5}\right)^3 = \left(x^{1/3}\right)^3$

$\frac{-1}{125} = x$

$(-2)^3 = \left(x^{1/3}\right)^3$

$-8 = x$

$u^2 + 11u + 10$

$(u + 1/5)(u + 10/2)$

$(5u + 1)(u + 2)$

$u = -1/5 \text{ & } -2$

Determine exact solutions-

$$x^4 - 2x^2 - 48 = 0$$

$$(x^2 - 8)(x^2 + 6) = 0$$

$$\sqrt{x^2} = \sqrt{8} \qquad \sqrt{x^2} = \sqrt{-6}$$

$$= \pm 2\sqrt{2} \qquad x = \pm\sqrt{6}i$$

Option.... Let $u = x^2$

$(x^2)^2 - 2x^2 - 48 = 0$

Since it's quartic with an x squared, we can use the "x" method and not u-substition. WE STILL CAN'T USE THE QUADRATIC FORMULA, HOWEVER. . .

$x - 13\sqrt{x} + 40 = 0$ ⟶ $u^2 - 13u + 40 = 0$

Let $x = u^2$ ←

Why?

Removes the radical....

$(u-8)(u-5)=0$

$u = 8 \text{ & } 5$

$X = u^2$

$X = 8^2$

$= 64$

$X = 5^2$

$= 25$

$x^{-2} - x^{-1} - 20 = 0$ ⟶ $(x^{-1})^2 - (x)^{-1} - 20 = 0$

Let $u = x^{-1}$

$u^2 - u - 20 = 0$

$(u-5)(u+4) = 0$

$u = 5 \text{ & } -4$

$(5)^{-1} = (x^{-1})^{-1}$

$\frac{1}{5} = x$

$(-4)^{-1} = (x^{-1})^{-1}$

$\frac{1}{-4} = x$

5^-1=.2 math→frac

. . .

$(x-5)^2 + 7(x-5) - 18 = 0$

Let u = x - 5

Rewrite-

$u^2 + 7u - 18 = 0$

$(u+9)(u-2) = 0$

$u = -9 \text{ & } 2$

$-9 = x - 5$

$-4 = x$

$2 = x - 5$

$7 = x$

$(x^2 - x)^2 - 14(x^2 - x) + 24 = 0$

Let u = ???? $x^2 - x$

$u^2 - 14u + 24 = 0$

$(u - 12)(u - 2) = 0$

$u = 12$ & 2

$12 = x^2 - x$

$0 = x^2 - x - 12$

$= (x - 4)(x + 3)$

$x = 4$ & -3

$2 = x^2 - x$

$0 = x^2 - x - 2$

$(x - 2)(x + 1)$

$x = 2$ & -1

$\{-3, -1, 2, 4\}$

-Suggested Practice-

Sec 1.6
page 179

42-58 evens

42. $\pm 3, \pm 2$

44. $\pm 1, \pm 3/2$

46. 36

48. $-\frac{1}{2}, \frac{1}{3}$

50. $-125, {}^{27}/_{8}$

52. $-243, 32$

54. 1

56. $-12, -1$

58. $-2, -1, 3, 4$

Simplify

$\sqrt{75}$ $\sqrt{-18}$

$\sqrt{32}$ $\sqrt{-72}$

$\sqrt{300}$ $\sqrt{-8}$

Examples

$\sqrt{20} = \sqrt{4 \cdot 5}$

$= 2\sqrt{5}$

$\sqrt{-50} = \sqrt{-1 \cdot 25 \cdot 2}$

$= 5i\sqrt{2}$

Determine u = ?

$(x-5)^2 - 4(x-5) - 21 = 0$

Let $u = x-5$

$x^4 - 5x^2 + 4 = 0$

none? use quadratic type x method

or

Let $u = x^2$

$x^{2/3} - 9x^{1/3} + 8 = 0$

Let $u = x^{1/3}$

$x^{-2} + 3x^{-1} + 2 = 0$

Let $u = x^{-1}$

$x + 3x^{1/2} - 10 = 0$

Let $u^2 = x$

$u^2 + 3(u^2)^{1/2} - 10 = 0$