

Sec 1.6.1

Solving higher-order polynomial equations by factoring

Solve-  $3x^4 = 27x^2$

$$3x^4 - 27x^2 = 0$$

$$3x^2(x^2 - 9) = 0$$

$$3x^2(x+3)(x-3) = 0$$

$$3x^2 = 0 \quad x+3=0 \quad x-3=0$$

$$x^2 = 0$$

$$x = 0 \quad x = -3 \quad x = 3$$

$$\{-3, 0, 3\}$$

- equation must = 0
- be written in descending order
- factor
- set factors = 0 and solve resulting equations

Do NOT divide through by variables!  
Solutions are "lost". Also, the variable could equal zero...can't divide by zero.

$$x^3 + x^2 = 4x + 4$$

$$(x^3 + x^2) + (-4x - 4) = 0$$

$$x^2(\cancel{x+1}) - 4(\cancel{x+1}) = 0$$

$$(x+1)(x^2 - 4) = 0$$

$$(x+1)(x+2)(x-2) = 0$$

$$x+1=0$$

$$x = -1$$

$$x+2=0$$

$$x = -2$$

$$x-2=0$$

$$x = 2$$

$$\{-2, -1, 2\}$$

Four terms?

Factor by

GROUPING

$$3x^4 - 24x = 0$$

$$3x(x^3 - 8) = 0$$

won't ever  
factor, imag  
solns

$$3x(x-2)(x^2+2x+4) = 0$$

$$3x = 0$$
$$x = 0$$

$$x - 2 = 0$$
$$x = 2$$

$$x = \frac{-2 \pm \sqrt{4 - 4(4)}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm \sqrt{-1 \cdot 4 \cdot 3}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$= -1 \pm \sqrt{3}i$$

4 solutions  
2 real  
2 imaginary

**Suggested Practice**  
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## Solutions

1.  $-4, 0, 4$
2.  $\pm 2, 0$
3.  $-2, -\frac{2}{3}, 2$
4.  $\pm \frac{3}{2}, 3$
5.  $\pm \frac{1}{2}, \frac{3}{2}$
6.  $\pm \frac{1}{3}, -1$
7.  $-2, \pm \frac{1}{2}$
8.  $\pm \frac{2}{3}, 2$
9.  $0, 2, 1 \pm i\sqrt{3}$  ←
10.  $0, 3, -\frac{3}{2} \pm i \frac{3\sqrt{3}}{2}$  ←

$$2x^4 = 16x$$

$$2x^4 - 16x = 0$$

$$2x(x^3 - 8) = 0$$

$$\downarrow$$
$$2x = 0$$
$$x = 0$$

$$\downarrow$$
$$x^3 - 8 = 0$$
$$x^3 = 8$$
$$x = 2$$

$$\left\{ \begin{array}{l} x^3 - 8 \\ (x-2)(x^2 + 2x + 4) \\ \vdots \end{array} \right.$$

$$x^3 - 8$$

$$(x-2)(x^2+2x+4)$$

$$x=2$$

$$x = \frac{-2 \pm \sqrt{4 - 4(4)}}{2}$$

$$= \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2}$$

$$= -1 \pm i\sqrt{3}$$

$$3x^4 - 81x = 0$$

$$3x(x^3 - 27) = 0$$

$$x=0 \quad (x-3)(x^2+3x+9)$$

$$x=3$$

$$x = \frac{-3 \pm \sqrt{9 - 4(9)}}{2}$$

$$= \frac{-3 \pm \sqrt{-27}}{2} \quad \left| \begin{array}{l} \text{red } -1 \cdot 9 \cdot 3 \\ \text{red } \rightarrow -3 \pm 3i\sqrt{3} \end{array} \right.$$
$$= \frac{-3}{2} \pm \frac{3i\sqrt{3}}{2}$$