Sec 3.3
Using division to determine zeros (roots, real solutions)

First, the Remainder Theorem states...if the polynomial f(x) is divided by x -c, then the remainder is f(c).

i.e. You can evalute a function using synthetic division. The remainder is the "y"/function value.

Use the remainder theorem to evaluate

$f(x) = x^3 - 4x^2 + 5x + 3$ at $f(2)$

$$
\begin{array}{r|rrrr}
2 & 1 & -4 & 5 & 3 \\
  &   & 2 & -4 & 2 \\
\hline
  & 1 & -2 & 1 & 5
\end{array}
$$

$f(2) = 5$

So, $f(2) = 5$

The Factor Theorem

$$\begin{array}{r} 4 \\ 3\overline{)12} \end{array}$$

If there is no remainder, then the divisor is a factor (or, in the case of synthetic division, a "zero") of the dividend.

Example-

$$\begin{array}{r} x+7 \\ x+3\overline{)x^2+10x+21} \end{array}$$

If there is no remainder, then $x+3$ is a factor of $x^2+10x+21$ and we know that $(x+7)(x+3) = x^2+10x+21$

$$2x^3-3x^2-11x+6 \div x-3$$

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -11 & 6 \\ \hline & & & & 0 \end{array}$$

There is no remainder so we know that 3 is a zero/root/solution to $2x^3-3x^2-11x+6$ and that $(x-3)$ is a factor of it.

Solve the equation $2x^3-3x^2-11x+6=0$ given that 3 is a zero/root.

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -11 & 6 \\ & & 6 & 9 & -6 \\ \hline & 2 & 3 & -2 & \big| \, 0 \end{array}$$

$(x-3)(2x^2+3x-2) =$
$2x^3-3x^2-11x+6$

$(x-3)(2x^2+3x-2)=0$

x^2+3x-4

$(x+\frac{4}{2})(x-\frac{1}{2})$

$(x-3)(x+2)(2x-1)=0$

$x=\{3,-2,\frac{1}{2}\}$

Suggested Practice
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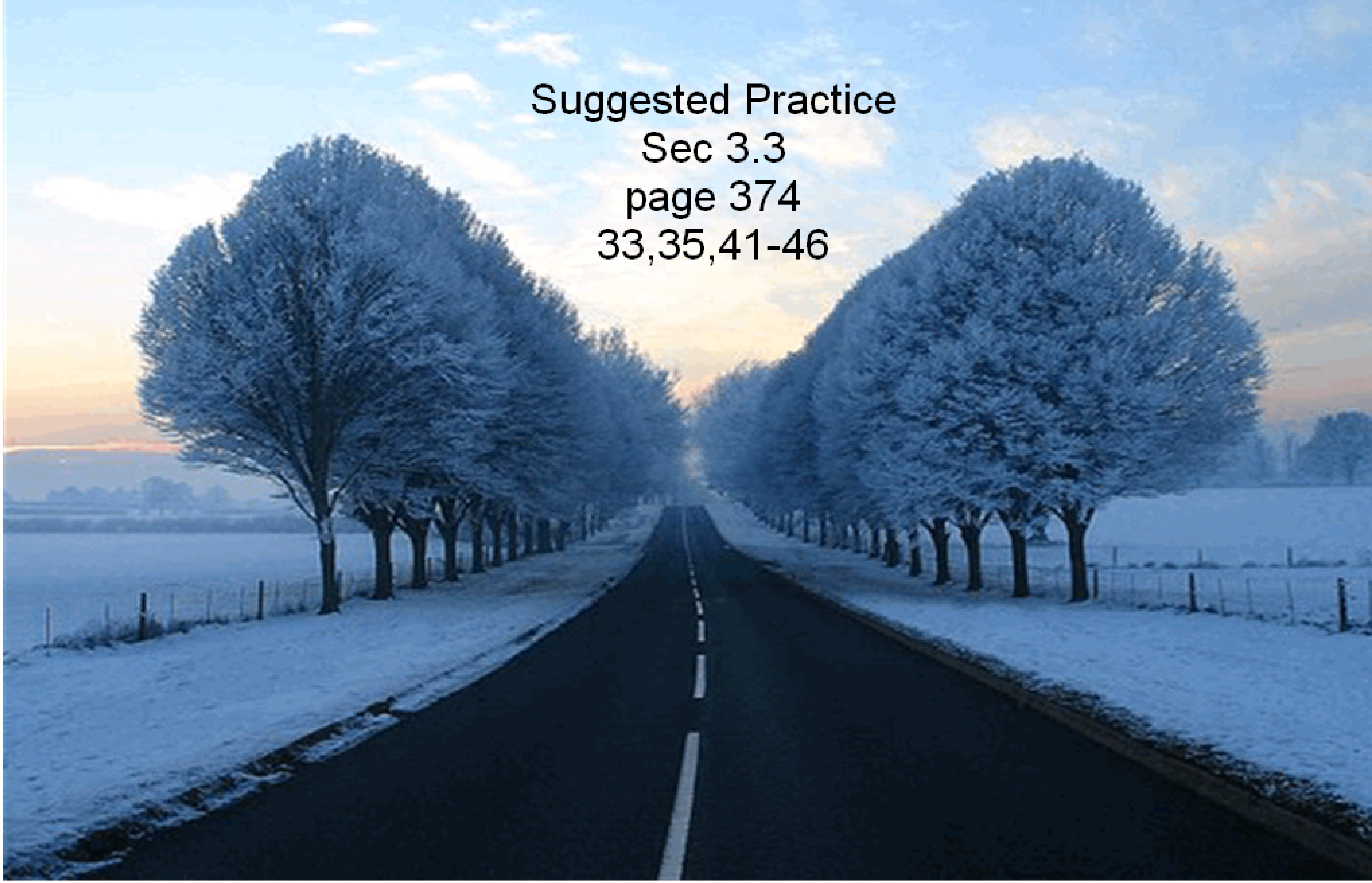

33. -25

35. -133

41. x^2-5x+6

$x=-1,2,3$

42. x^2-3x+2

$x=\pm 1,2$

43. $\{-1/2,1,2\}$

44. $\{-2,1/2,3\}$

45. $\{-3/2,-1/3,1/2\}$

46. $\{-4,-1/3,2\}$