Sec 3.2
Polynomial Functions & Graphs
Today- end behavior

A **polynomial** is a **single term or the sum of two or more terms** containing variables with **whole number exponents** .

Consider the polynomial: $3x^4 - 2x^3 - 5x + 6$

This polynomial contains four terms. **It is customary to write the terms in order of descending powers of the variable.** This is the standard form of a polynomial. Here are two other polynomials which are written in standard form.

$$5x^3 - 7x^2 + 2x - 8$$

$$8x^4 - 3x + 6$$

The **degree of a polynomial** is the greatest degree of any term of the polynomial. The degree of a term $ax^n y^m$ is $(n+m)$

$4x^3y^5$

degree = 8

and the **coefficient** of the term is *a*. If there is exactly one term of greatest degree, it is called the **leading term**. It's coefficient is called the leading coefficient. Consider the polynomial: $3x^4 - 2x^3 - 5x + 6$

3 is the leading coefficient. The degree is 4.

- The **degree of a polynomial** is the degree of its highest order term.
- Example:

Degree 3 Polynomial: $5x^3 - 7x^2 + 2x - 8$

Degree 4 Polynomial: $8x^4 - 3x + 6$

Also known as linear (1), quadratic (2), cubic (3), quartic (4)

not assessed, but good to know...

- Monomial: A polynomial with one term.
- Binomial: A polynomial with two terms.
- Trinomial: A polynomial with three terms.

Example: $8x^4 - 3x + 6$

This is a 4th degree trinomial.

Not a polynomial

x cannot be inside a radical

$$N(x) = 2x^2 + \sqrt{x}$$

No negative exponents allowed

$$N(x) = 3x^3 + 4x^{-2} + 3$$

$$N(x) = 8x^2 - \frac{2}{x}$$

$2x^{-1}$

x cannot be in the denominator

x cannot be an exponent

$$N(x) = 4^x + 3$$

Exponent cannot be a fraction

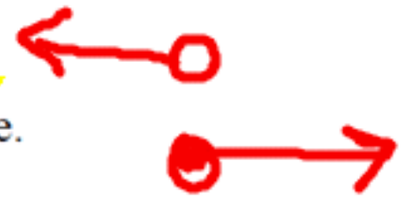
$$N(x) = 5x^{1/3} + 3x^2 - 2$$

Graphs of Polynomial Functions

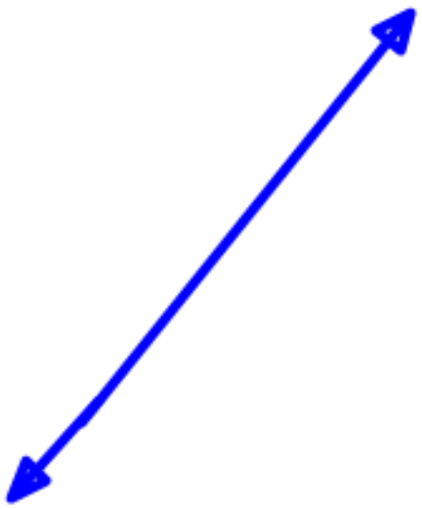
Polynomial functions of degree 2 or higher have **graphs** that are ***smooth*** **and** ***continuous*** .

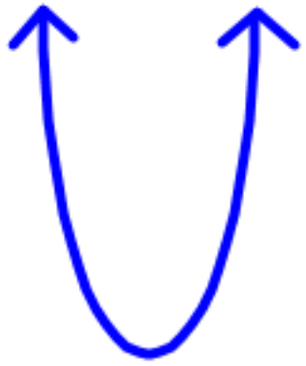
By **smooth** , we mean that the graph contains only rounded corners with no sharp corners.

By **continuous** , we mean that the graph has no breaks and can be drawn without lifting the pencil from the page.

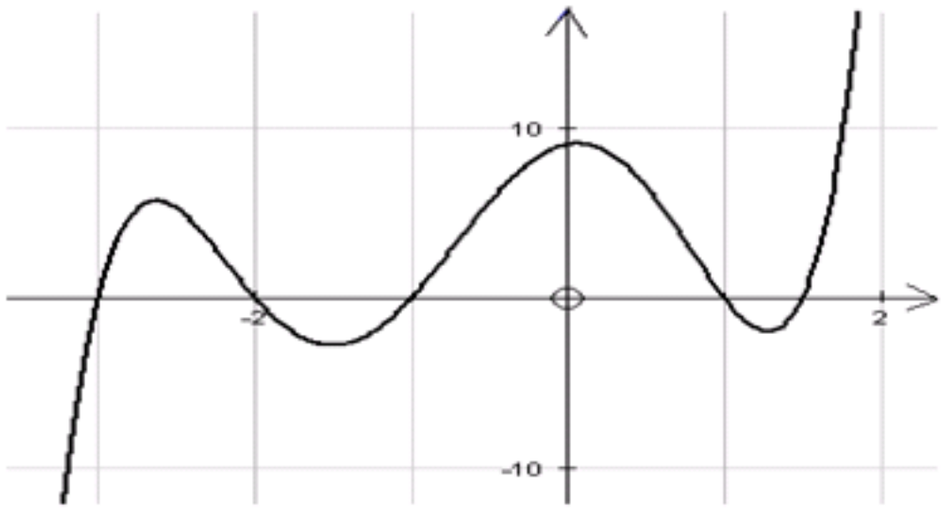


Examples-

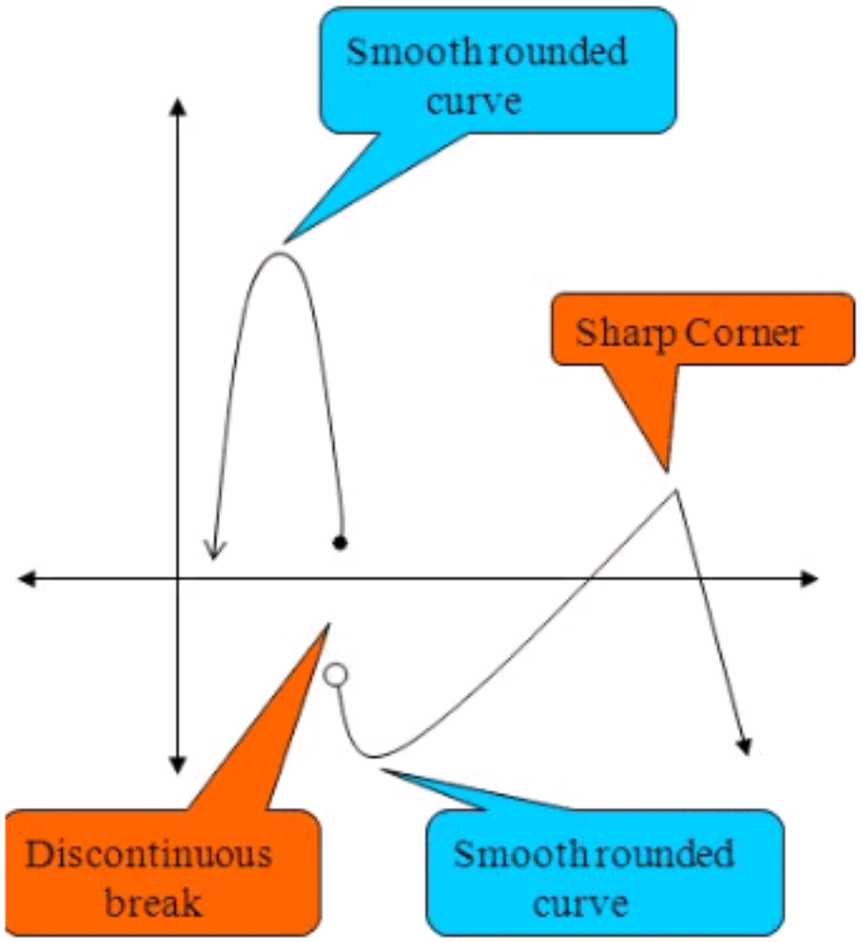




$f(x) = x^5 + 3.5x^4 - 2.5x^3 - 12.5x^2 + 1.5x + 9$

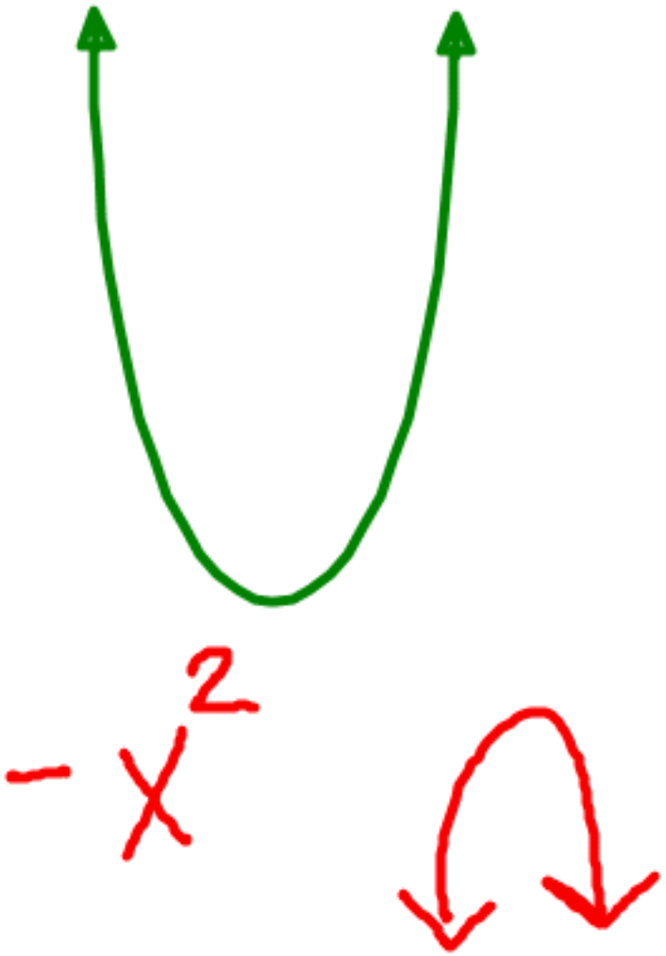


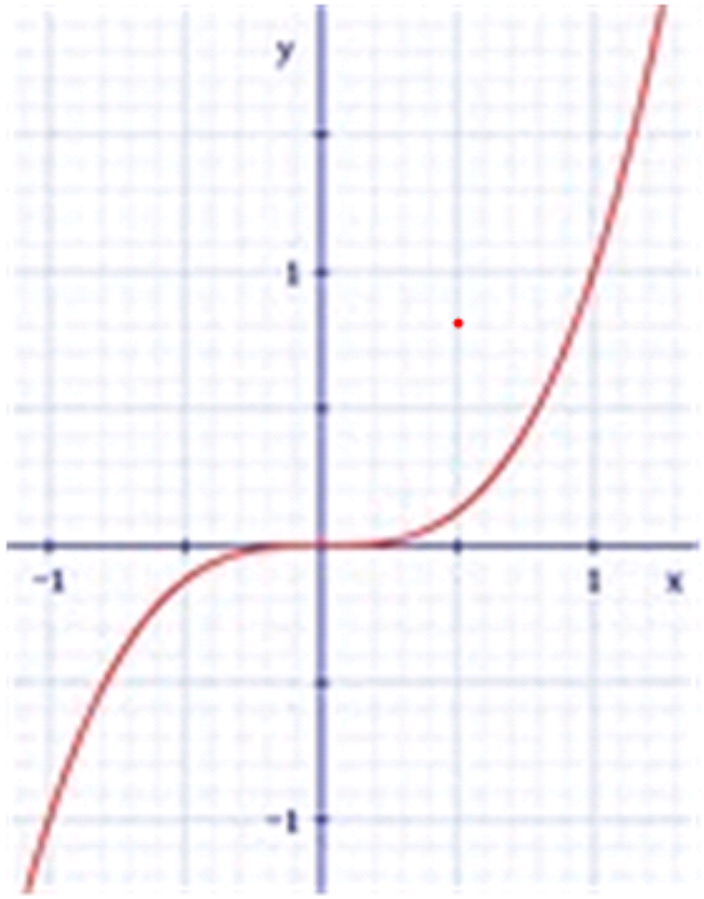
www.analyzemath.com



The graph below does *not* represent a *polynomial* function. Although it has a couple of smooth, rounded corners, it also has a sharp corner and a break in the graph. Either one of these last two features disqualifies it from being a *polynomial* function.

Recall what $y = x^2$ and $y = x^3$ look like...





Generalize the same end behavior to all even degree or odd degree polynomials.

Recall- x^2 x^3

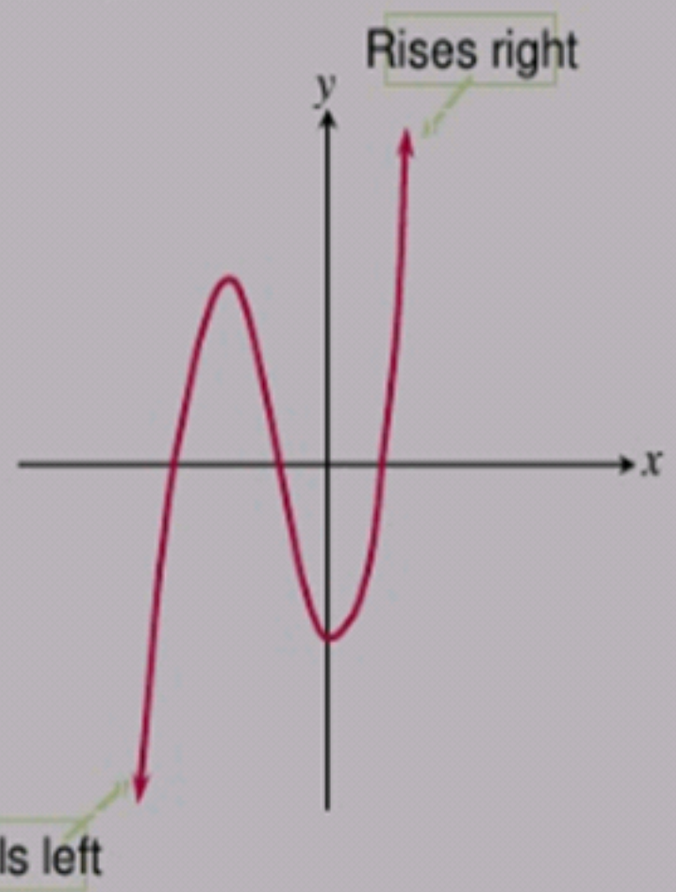
	Even degree	Odd degree
Pos coeff.	$x \to -\infty$, $y \to \infty$ and $x \to \infty$, $y \to \infty$	$x \to -\infty$, $y \to -\infty$ and $x \to \infty$, $y \to \infty$
Neg coeff.	$x \to \infty$, $y \to -\infty$ and $x \to \infty$, $y \to -\infty$	$x \to -\infty$, $y \to \infty$ and $x \to \infty$, $y \to -\infty$

Leading Coefficient Test for End Behavior

- Even-degree polynomial functions have graphs with the ***same behavior at each end.***
- Odd-degree polynomial functions have graphs with ***opposite end behavior.***

Use the Leading Coefficient Test to determine the end behavior of the graph of Graph the quadratic function $f(x) = x^3 + 3x^2 - x - 3$.

Solution Because the degree is odd ($n = 3$) and the leading coefficient, 1, is positive, the graph falls to the left and rises to the right, as shown in the figure.



Use the Leading Coefficient Test to determine the end behavior of the graph of

$f(x) = -3x^3 - 4x + 7$

think

x^3 $-x^3$

rises to left

falls to right

$5x^6+2x$

think...x^2

rising to the left & right

$-3x^5+4x^2-7$

think...$-x^3$

rises to left
falls to right

$.2x^7+4x$

think.... x^3

falls to left
rises to right

LC +
deg even

LC +
deg odd

LC −
deg odd

LC −
deg even

Describe what type of LC yields these end behaviors...

Suggested Practice
Sec 3.2
page 360
1-24 all

Solutions-

1. is, 3
2. is, 4
3. is, 5
4. is, 7
5. is not
6. is not
7. is not
8. is not
9. is not
10. is, 2
11. is
12. not
13. not
14. is
15. b
16. c
17. a
18. d
19. down to left
    up to right
20. same as 19
21. up on both ends
22. same as 21
23. down on both ends
24. same as 23